MATHEMATICAL DESCRIPTION OF SOLID-PHASE ENTRAINMENT DURING FLUIDIZATION IN STEADY AND UNSTEADY CONDITIONS

O. M. Todes and O. B. Tsitovich

Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 5, pp. 609–615, 1965

UDC 541.182

An examination is made of the dynamic structure of solid-phase equilibrium during fluidization both in steady and in unsteady conditions. A mathematical description is obtained for the entrainment of the solid phase. The two-stage nature of the process (dense phase–sublayer space–entrainment) explains the inflection in the entrainment curves.

During fluidization of a solid granular material by an ascending gas stream, some portion of the solid phase is unavoidably carried away with the stream. The main part of this loss consists, as a rule, of the finest grains—dust. We deem it necessary to commence an analysis of the mechanism and laws of entrainment with an examination of the general structure of a fluidized bed as a whole.

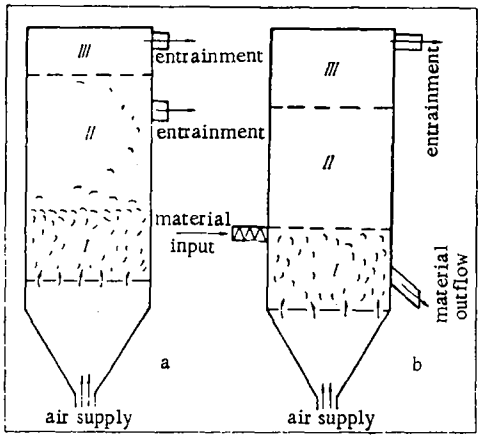

Fig. 1. Entrainment during periodic (a) and continuous (b) fluidization.

The outline diagram of a reactor with a steady fluidized bed is shown in Fig. 1a. In general, the reactor may be divided into three main regions. Region 1—the basic fluidized bed—is located immediately above the grid which supports the bed and distributes the gas. The mean bed density in this region is close to the bulk density of the bed at rest, and gradually decreases with increase of gas stream velocity. Throughout the whole inhomogeneity of the fluidized bed the mean local density varies comparatively little in parts of this region which differ according to the height and diameter of the reactor [1, 2].

In the upper part of the basic fluidized bed we pass into Region II—the ejection region or separation zone. As a result of the outburst of bubbles and the upsurge of "parcels" of grains of the solid phase, particles are thrown up from the bed to a certain height and then fall back. Because of the random distribution of kinetic energy of the outgoing particles and of the heights to which they are thrown, the distribution of mean density with height in Region II is exponential in nature:

$$\rho = \rho_0 \exp(-z/h_0). \quad (1)$$

The quantity h_0 is very small as a rule, and the mean density in Region II falls very sharply with height. At a large flow velocity, however, this region may extend over the whole reactor.

When there is a large height of reactor above the separation Region II, there exists Region III, which is reached only by very fine-grained particles, whose critical velocity ω_s is less than the flow velocity ω_0, calculated over the whole reactor section. In this region pneumatic transport of dust occurs at a very small concentration which is practically unchanging with height.

If the gas outlet from the reactor is situated in Region II (Fig. 1a), then entrainment of the solid phase will be observed, the concentration depending on the height in accordance with (1). This kind of entrainment will occur also in a monodisperse fluidized bed containing grains of identical size. For a polydisperse fluidized bed with flow velocities very close to the critical velocity, separation with height is observed, and the finest (dust) particles will be preferentially entrained. When the flow velocity is increased, however, because of intensive mixing of the solid phase in Regions I and II, the fractional composition of mechanical entrainment will not be significantly different from that of the basic fluidized bed.

Finally, for certain highly dispersed materials (polychlorovinyl resin, shale flotation concentrate), "parcels" of whose granules are quite stable in the bed (Region 1) and disintegrate only in Region II, the change in concentration in Region 2 may be prove to be insignificant, and conditions may approximate to those of Regions III.

When there is continuous charge into and discharge from the reactor (for example, of coking catalyst in catalytic cracking), a steady flow distribution of solid is established, as shown in Fig. 1b. A definite balance is established between supply, discharge, and entrainment, both for the total mass and for the separate granule fractions of the bed.

Leaving aside the perfectly monodisperse bed (for example, of spheres of identical diameter), the finest granules are involved in entrainment. Granules which are so large that the mean height of their ejection h_0 is several times less than the height of the separation zone h_c, will not be carried away from the reactor

with the stream. But grains of diameter d, less than some critical value d_m, will have a varying probability of becoming entrained, the probability increasing with reduction of d. It is therefore expedient to consider as a frist approximation a bidisperse model of the fluidized bed the main mass of which consists of larger granules with $d > d_m$, which are not carried off from the reactor, with a small portion of "fines" with $d < d_m$ which may be blown away from the bed.

We shall consider a steady fluidized bed with no specific supply and discharge of solid. We shall designate by N_1 the total number of fine grains in the basic fluidized bed of height l_0, and by N_2 the total number of fine grains in the separation volume at any time τ. There is intensive mixing in the first region, and the local concentration of fines $n_1 = N_1/l_0S$ is the same throughout the whole volume of the basic fluidized bed.

In the separation volume the concentration of fines falls exponentially with height:

$$n_2(z) = n_2(0)\exp(-z/h_0). \qquad (2)$$

The constant $n_2(0)$ is determined from the normalizing condition

$$N_2 = S\int_0^{l_c} n_2(z)\,dz, \qquad (3)$$

and finally

$$n_2(z) = \frac{N_2\exp(-z/h_0)}{h_0S\{1-\exp(-l_c/h_0)\}}. \qquad (4)$$

Transfer will occur by particles crossing the boundary between the regions, the rate of transfer being proportional to the cross section of the reactor and the concentrations n_1 and n_2.

The balance of fine grains in the basic bed will be determined by the differential equation

$$\frac{dN_1}{d\tau} = -a_1Sn_1 + a_2Sn_2(0) = -\frac{a_1}{l_0}N_1 + \frac{a_2N_2}{h_0\{1-\exp(-l_c/h_0)\}},$$

where the coefficients of proportionality a_1 and a_2 depend on the hydrodynamic properties of the bed. Putting

$$a_1/l_0 = \lambda_1;\ a_2\{h_0\{1-\exp(-l_c/h_0)\}\}^{-1} = \lambda_2, \qquad (5)$$

we finally obtain

$$\frac{dN_1}{d\tau} = -\lambda_1N_1 + \lambda_2N_2. \qquad (6)$$

The small particles entering the separation volume are in part turned back with velocity λ_2N_2, and in part carried away from the reactor through the upper boundary. Thus the balance equation for the fine grains in the separation space has the form

$$\frac{dN_2}{d\tau} = \lambda_1N_1 - \lambda_2N_2 - \lambda_3N_2, \qquad (7)$$

where

$$\lambda_3 = a_3\exp(-l_c/h_0)\{h_0\{1-\exp(-l_c/h_0)\}\}^{-1}. \qquad (7')$$

If we denote by $N_3 = N_0 - N_1 - N_2$ the number of fine grains carried out of the reactor up to time τ, then a third balance equation

$$\frac{dN_3}{d\tau} = \lambda_3N_2 \qquad (8)$$

is a simple consequence of the two preceding equations.

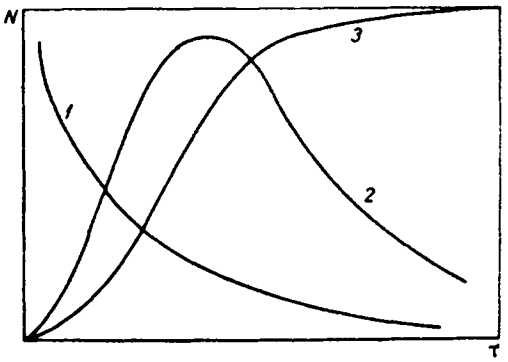

Fig. 2. Typical form of the time dependence of the numbers of grains in the regions and entrained: 1) N_1; 2) N_2; 3) N_3.

In the case of a tall reactor with a pneumatic transport region, Region III, particles whose critical velocity is less than the stream velocity are carried away from the reactor. For these particles $h_0 \to \infty$ and $\lambda_2 \to a_2/l_c$; $\lambda_3 \to a_3/l_c$, the balance equations are formally the same as the system (6), (7).

In the case of very slow entrainment, when $\lambda_3 \ll \lambda_2; \lambda_1$, a quasisteady equilibrium between Regions I and II is established very quickly after the start of the process:

$$\lambda_1N_1 \approx \lambda_2N_2;\ N_2 \approx \lambda_1N_1/\lambda_2;\ N_3 \approx N_0 - (\lambda_1+\lambda_2)N_1/\lambda_2.$$

Substituting these relations into (8), we obtain

$$\frac{dN_1}{d\tau} \approx -\lambda_3\lambda_1N_1/(\lambda_2+\lambda_1), \qquad (9)$$

i.e., the monomolecular law introduced by Leva [3].

In the general case with congruent values of the parameters, the solution of the original system of differential equations (6)–(8) with the initial condition

$$N_1(0) = N_0;\ N_2(0) = N_3(0) = 0$$

has the form

$$N_1(\tau) = \frac{N_0}{k_1-k_2}\times \times\{(\lambda_1-k_2)\exp(-k_1\tau) - (\lambda_1-k_1)\exp(-k_2\tau)\}, \qquad (10)$$

$$N_2(\tau) = \frac{\lambda_1N_0}{k_1-k_2}\{\exp(-k_2\tau) - \exp(-k_1\tau)\}, \qquad (11)$$

$$N_3(\tau) = \frac{\lambda_1\lambda_3 N_0}{k_1 - k_2} \times \times \left\{\frac{1-\exp(-k_2\tau)}{k_2} - \frac{1-\exp(-k_1\tau)}{k_1}\right\}, \quad (12)$$

where k_1 and k_2 are the roots of the quadratic equation

$$k_{1,2} = \frac{\lambda_1+\lambda_2+\lambda_3}{2} \pm \sqrt{\left(\frac{\lambda_1+\lambda_2+\lambda_3}{2}\right)^2 - \lambda_1\lambda_3}.$$

The typical shape of the curves of (10)–(12) is shown in Fig. 2. The initial parts of these curves are described by the approximate equations

$$N_1 \approx N_0 - \lambda_1 N_0 \tau;\ N_2 \approx \lambda_1 N_0 \tau, \quad N_3 \approx (\lambda_1\lambda_3/2) N_0 \tau^2. \quad (13)$$

After a certain time interval $\tau_m = (\ln k_1 - \ln k_2)/(k_1 - k_2)$, the concentration of fines in the space above the bed reaches a maximum. At the same time, the rate of entrainment $dN_3/d\tau$ reaches a maximum, while a point of inflection is observed on the curve $N_3(\tau)$. As has been noted above, when $\lambda_3 \ll \lambda_2$; λ_1 a comparatively insignificant fraction of the total fines will have been carried off up to time τ_m:

$$N_{3,m}/N_0 = \ln\frac{(\lambda_1+\lambda_2)^2}{\lambda_1\lambda_3}\left[\frac{(\lambda_1+\lambda_2)^2}{\lambda_1\lambda_3}\right]^{-1},$$

and after a further time (2–3) τ_m, the entrainment will proceed practically in accordance with the first-order equation

$$N_3(\tau) \approx N_0\left\{1-\exp\left(-\frac{\lambda_1\lambda_3}{\lambda_1+\lambda_2}\tau\right)\right\}. \quad (14)$$

It follows from comparison of expressions (5) and (7') that, as the height h_c of the separation space increases, other conditions being equal (constant composition of the original solid, constant gas flow rate, etc.), the entrainment slows up greatly, and the quantitative law (12) approximates to the first-order equation (14).

The dependence on gas flow rate ω proves to be more complicated. Firstly, with increase of ω, entrainment of larger particles becomes possible, and the boundary d_m shifts. If we consider that because of the exponential nature of (1), this boundary is determined by the approximate condition $h_0(d_m) \approx l_c/4$, then the influence of the stream velocity is determined by the fact that as the energy of fluctuating motion in the fluidized bed increases, so does the mean ejection height $h_0(d)$. Secondly, as ω increases, the mean ejection height h_0 of the fractions being entrained must increase. Since, according to (7'), this quantity enters exponentially into the probability of ejection (roughly in the same way as temperature enters into an expression for chemical reaction rate), then the entrainment must increase greatly as it increases. It is evident that the power relationship of type $\lambda_3 \sim \omega^n$ with large exponents ($n \approx 4$) introduced in [3–10] are only attempts to select an interpolation formula for the exponential function (7'), and have limited significance.

In the case of a reactor with a fluidized bed having continuous circulation of solid, the rate of entrainment will be constant. We shall denote by $c = \text{const}$ the number of fine grains supplied to the apparatus in unit time, this being a certain fraction $p \ll 1$ of the total amount of material supplied. We shall denote the rate of discharge by $\lambda_0 N_1$, while N_4 will be the number of fine grains discharged from the apparatus after a certain time τ. Then the balance equation for the fines takes the form

$$\frac{dN_1}{d\tau} = -\lambda_1 N_1 + \lambda_2 N_2 + c - \lambda_0 N_1,$$
$$\frac{dN_2}{d\tau} = \lambda_1 N_1 - \lambda_2 N_2 - \lambda_3 N_2, \quad (15)$$
$$\frac{dN_3}{d\tau} = \lambda_3 N_2,\ \frac{dN_4}{d\tau} = \lambda_0 N_1.$$

When the process is steady, the number of fines in the bed and in the separation volume will cease to vary, i.e.,

$$\frac{dN_1}{d\tau} = \frac{dN_2}{d\tau} = 0. \quad (16)$$

From (15) and (16) we can find the steady concentrations

$$N_1 = (\lambda_2+\lambda_3)\frac{c}{\lambda_0}\left(\lambda_2 + \frac{\lambda_1+\lambda_0}{\lambda_0}\lambda_3\right)^{-1},$$
$$N_2 = \lambda_1\frac{c}{\lambda_0}\left(\lambda_2 + \frac{\lambda_1+\lambda_0}{\lambda_0}\lambda_3\right)^{-1}, \quad (17)$$

the rate of entrainment

$$\frac{dN_3}{d\tau} = \lambda_3\lambda_1\frac{c}{\lambda_0}\left(\lambda_2 + \frac{\lambda_1+\lambda_0}{\lambda_0}\lambda_3\right)^{-1}, \quad (18)$$

and the rate of removal of fines from the apparatus

$$\frac{dN_4}{d\tau} = (\lambda_2+\lambda_3)c\left(\lambda_2 + \frac{\lambda_1+\lambda_0}{\lambda_0}\lambda_3\right)^{-1}. \quad (19)$$

If there were no entrainment, then $\lambda_3 = 0$, and the concentration of fines in the apparatus:

$$N_1^0 = c/\lambda_0 \quad (20)$$

would comprise the same portion, p, as in the original material. Comparing (20) and (17), we see that when entrainment occurs, the absolute concentration of fines in the apparatus and in the material carried away comprises a fraction

$$\frac{N_1}{N_1^0} = \frac{1+\lambda_3/\lambda_2}{1+(1+\lambda_1/\lambda_0)\lambda_3/\lambda_2} < 1 \quad (21)$$

of p. This quantity may be measured in an experiment as the ratio of the amount of fines carried away to the amount introduced in to the apparatus.

From the ratio of the rate of entrainment to rate of removal

$$\lambda_0 \frac{dN_3}{d\tau} \Big/ \frac{dN_4}{d\tau} = \frac{\lambda_1\lambda_3}{\lambda_2+\lambda_3} = \text{const}, \qquad (22)$$

we may obtain a single equation relating the three constants λ_1, λ_2, λ_3. It can be seen from (9) and (14) that the ratio also appears in the rate of entrainment during periodic operation of the process without circulation of solid through the reactor.

If we measure the ratio of the concentration in the bed to that in the space above it under steady and quasisteady conditions, we may determine the quantity

$$N_2/N_1 = \lambda_1/(\lambda_2 + \lambda_3) = \text{const}. \qquad (23)$$

From the data of (22) and (23) we can find the main parameter determining the rate of removal, i. e., the quantity λ_3.

To find all three constants it is still necessary to measure the concentrations with a very tall reactor, when $\lambda_3 = 0$. This gives a third equation

$$N_{2,\infty}/N_{1,\infty} = \lambda_1/\lambda_2. \qquad (24)$$

Thus, entrainment must be intimately connected with the structure of a fluidized bed and its parameters. The model developed above for mathematical analysis of the kinetics of entrainment is, of course, a first approximation to reality. Comparison of it with experiment will permit the evaluation of the basic kinetic constants λ_i and their dependence on the conditions of operation of the process. For widely polydisperse granular materials it may be necessary to refine the treatment and even to analyze the separate fractions.

NOTATION

ρ and ρ_0) mean and initial density of two-phase system in the space above the bed; h_0) mean height of rise of ejected particles; z) height, measured from the top of Region I; ω_s and ω_0) critical velocity of fine particles and gas flow velocity, relative to the whole cross section of the apparatus; l_c) height of the separation none (Region II); d) diameter of the solid grains; d_m) diameter of solid grains for which $\omega_s = \omega$; l_0) height of the bed itself (Region I); S) cross-sectional area of the reactor; λ_1, λ_2, λ_3) kinetic constants of entrainment.

REFERENCES

1. N. I. Syromyatnikov and V. F. Volkov, Processes in a Fluidized Bed [in Russian], Metallurgizdat, Sverdlovsk, 1959.

2. N. I. Syromyatnikov, DAN SSSR, 93, no. 3, 1953.

3. M. Leva, Fluidization [Russian translation], ed. N. I. Gel'perin, Gostoptekhizdat, 1961.

4. M. I. Fridland, Khimiya i tekhnologiya topliv i masel, no. 7, 1962.

5. M. I. Fridland, Khimicheskoe mashinostroenie, no. 4, 1962.

6. M. I. Fridland and A. I. Skoblo, Khimicheskoe mashinostroenie, no. 5, 1960.

7. D. B. Thomas, P. I. Grey, and S. B. Watkins, Brit. Chem. Engng., 6, no. 2, 1960.

8. Wen Chim Jung and R. F. Hashinger, A. I. Ch. E. Jr.; 6, no. 2, 1960.

9. F. A. Zenz and N. A. Weil, A. I. Ch. E. Jr., 4, no. 4, 1958.

10. G. L. Osberg and D. H. Sharlswarth, Chem. Engng. Progr., 27, no. 11, 1951.

12 January 1965

Higher Military College of Engineering and Technology, Leningrad